Danial Elias

$x_1 = 19$

$\rightarrow \sum_{i=1}^{n} (2i-1) = n^2$

$\rightarrow$ Plugging in 5 Values $\Rightarrow$ $1 + 3 + 5 + 7 + 9 \ldots + (2n-1) = n^2$

$\rightarrow$ Solution ① $\Rightarrow n=1 \rightarrow 2(1)-1 = 1$

$\boxed{1 = 1 \checkmark}$

② Assume $1 + 3 + 5 + 7 + 9 \ldots + (2k-1) = k^2$ for $k \geq 1$

③ proof for $k = n+1$

$$\rightarrow \underbrace{1 + 3 + 5 + 7 + 9 \ldots + (2k-1)}_{= k^2} + (2(k+1)-1) = (k+1)^2$$

So, $k^2 + (2(k+1)-1) =$

k^2 + 2k + 2 - 1 = k^2 + 2k + 1 = (k + 1)^2
Done

(1) proof $20 \mid 3^{8n+3} - 7$ for every $n \geq 1$

(1) $n = 1 \rightarrow 20 \mid (3^{11} - 7) \Rightarrow$ yes 20 is a factor of $(3^{11} - 7)$

(2) Assume $20 \mid 3^{8n+3}$ is valid for some $n \geq 1$

(3) proof it for $(n+1)$

$$\rightarrow 20 \mid 3^{8(n+1)+3} - 7$$

$$\rightarrow 20 \mid 3^{(8n+8)+3} - 7$$

$$3^{(8n+8)+3} - 7 \rightarrow 3^{8n+3} \cdot 3^8 - 7 + 3^8 \cdot 7 - 3^8 \cdot 7$$

$$\frac{3^{(8n+3)+8} - 7}{20} \rightarrow \frac{3^8(3^{8n+3} - 7)}{20} + \frac{3^8 \cdot 7 - 7}{20}$$

$\rightarrow$

$* \; 20 \mid 3^8(3^{8n+3} - 7)$ according to step two.

$* \; 20 \mid 3^8 \cdot 7 - 7$, From the calculator.

$\boxed{\text{Hence, } 20 \text{ is a factor of } 3^{8n+3+8} - 7}$